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Report on the Progress made in the Calculations for a New Method of treating the Lunar Theory. By Sir G. B. Airy, K.C.B., Astronomer Royal.

In the *Monthly Notices* of this Society for 1874, January, I explained the form which I proposed to give to a New Method of treating the Lunar Theory, and I stated that I had actually commenced some of the numerical developments of which the new method mainly consists. In my usual Annual Report to the Board of Visitors of the Royal Observatory, I mentioned (in the ordinary course of report) that I had arranged such a method of treating the Theory, that the principal part of the work was purely numerical, and that a computer had been engaged, at my expense, in proceeding with some parts of the work. The Board spontaneously passed a resolution to the effect that this undertaking might properly be considered as a part or extension of the astronomical calculations made at the Observatory, and that in their opinion it might properly be undertaken at the public expense. I submitted this opinion of the Board of Visitors to the Admiralty, through whom it was placed before the Treasury; and, after explanation of the nature and object of the proposed work, I received authority from the Treasury for incurring the expense required for carrying on the computations through the current Financial year. I have subsequently submitted an estimate for progress of work during the year 1875–1876. Two computers have been employed, the aggregate of their times of employment to 31st December 1875 being about 10 months.

We have had painful evidence of the difficulties that may arise in utilising a laborious investigation, when by some sad chance the possibility of its completion by the original proposer has been suddenly terminated, and scarcely sufficient explanation has been left to enable successors to enter into the remaining steps in the same spirit as that which guided the primary operations. It has appeared to me, therefore, desirable in prosecuting a new theory (which, though far simpler than preceding theories in the intellectual labour demanded, is still sufficiently extensive and sufficiently novel in form to require some study before a following astronomer could actively take it up), to place on record such a description of the method of proceeding, and, from time to time, such a statement of the advance of the work, as will enable a successor, if necessary, to carry on the operations without much difficulty to their close.

1. Referring to my paper "Proposed New Method &c." already cited, paragraph marked VI., it will be seen that the first step is to convert Delaunay's sexagesimal angles, expressed in seconds of arc, into equivalent expressions in decimal fractions

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of radius; that these expressions are to be limited to the seventh decimal place, of which the unit is 10^{-7}; and that, this limit being understood throughout, trouble and confusion will be avoided by omitting decimal cyphers preceding the effective figures. To convert the seconds of arc into decimals of radius, it is necessary to multiply the number of seconds by 0·000004848137. Therefore, multiplying by 100000, and considering the final figure of Delaunay's coefficients to be the second decimal of the sexagesimal second, we have to arrange a method, applicable to all coefficients, by which 10000000 may be converted into 4848137. The method adopted is one analogous to that known in books of arithmetic by the term "Practice;" it never leaves a doubt on the value of the leading figure, and it employs no unnecessary figures. By trial, the following steps are adapted for this case; every succeeding number being formed from the preceding number by use of the fraction in the margin:

		10000000	
$+\frac{1}{2}$	=	+ 5000000	
$-\frac{3}{100}$	=		− 150000
$-\frac{1}{90}$	=		− 1667
$-\frac{1}{8}$	=		− 208
$+\frac{1}{20}$	=	+ 10	
$+\frac{1}{5}$	=	+ 2	
		+ 5000012	− 151875
		− 151875	
		+ 4848137	

In the application to actual coefficients, the last term is never wanted, and the preceding term only once. Of the numbers, which I now proceed to exhibit as specimens, the first is the largest coefficient in longitude, the second is one taken from the middle of the series:—

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	$22639''\cdot06$	
$+\frac{1}{2}=$	$+1131953$	
$-\frac{3}{100}=$		-33959
$-\frac{1}{90}=$		-377
$-\frac{1}{8}=$		-47
$+\frac{1}{20}=$	$+2$	
	$+1131955$	-34383
	-34383	
	$+1097572$	

	$36''\cdot16$	
$+\frac{1}{2}=$	$+1808$	
$-\frac{3}{100}=$		-54
$-\frac{1}{90}=$		-1
	$+1808$	-55
	-55	
	$+1753$	

In succeeding parts of the work, several thousands of multiplications are performed in the same manner; the numbers to be multiplied being arranged in series, of which every term has the same multiplier; and the marginal fractions for that multiplier being determined by trial.

In some cases it is necessary to perform division; thus, in order to reduce the expression for parallax from the form

$$\text{Parallax} = 3422\cdot700 + \text{series of terms}$$

to the form

$$\frac{a}{r} = \{1 + \text{new series of terms}\},$$

it is necessary to determine by trial a process which will convert 34227000 into 10000000. The marginal fractions are found to be

$$+\frac{1}{3},\quad -\frac{1}{9},\quad -\frac{1}{10},\quad -\frac{1}{9},\quad -\frac{1}{30}.$$

2. The number of coefficients of cosines of different arguments in the expression for Parallax, adopted from Delaunay, and converted by the process above described, is 100. The number of coefficients of sines of different arguments in the expression for Longitude, is 170; and that in the expression for Latitude, is 142; which are similarly converted. By subsequent operations, the numbers of coefficients in squares, &c., of these series are raised to 148, 170, 180. The smallest value retained is 1 (in my notation), or a number not less than its half.

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3. The great mass of operations consists in multiplying together two series, such as

$$A + B\cos\boxed{P} + C\cos\boxed{Q} + D\cos\boxed{R} + \text{etc.}$$

$$a + b\cos\boxed{p} + c\cos\boxed{q} + d\cos\boxed{r} + \text{etc.}$$

The coefficients of the upper line are all multiplied by a (by the process above described); then by b, and so on. The general product is

$$Aa + Ba\cos\boxed{P} + Ca\cos\boxed{Q} + \text{etc.} + Ab\cos\boxed{p} + \frac{Bb}{2}\cos\boxed{P+p} + \frac{Bb}{2}\cos\boxed{P-p}$$

$$+ \frac{Cb}{2}\cos\boxed{Q+p} + \frac{Cb}{2}\cos\boxed{Q-p} + \text{etc.} + Ac\cos\boxed{q} + \frac{Bc}{2}\cos\boxed{P+q}$$

$$+ \frac{Bc}{2}\cos\boxed{P-q} + \frac{Cc}{2}\cos\boxed{Q+q} + \frac{Cc}{2}\cos\boxed{Q-q} + \text{etc.}$$

In the great majority of instances, the new arguments formed in these multiplications are to be found in each of the series which are multiplied together; and the form of the series produced by the collection of these products is the same as that of the first series, with the addition, perhaps, of a few small terms. In the primary series for Parallax, Longitude, and Latitude, the coefficients are arranged in the order of descending magnitude, and this order is little disturbed in the operations of multiplying series together.

4. The coefficients of $\frac{a}{r} - 1$ are represented by the ordinal numbers (written in red ink) 1, 2, 3, &c., to 100; those of $\left(\frac{a}{r} - 1\right)^2$ by 201, 202, &c., to 348, the small figures following centuries being attached to the same cosines, as far as they go: those of $\left(\frac{a}{r} - 1\right)^3$ by 401, 402, &c., to 535, the small figures following centuries being attached to the same cosines, &c. The coefficients of latitude are represented by 2401, 2402, &c., those of longitude by 5001, 5002, &c.; and the coefficients of products and other functions formed from them, by different series of similar order, the first always being a century increased by 1.

5. When powers of $\frac{a}{r} - 1$, to the fifth power, have been formed by successive multiplications by the series for $\frac{a}{r} - 1$, (forming a new series for each power, of which the terms are distinguished by new centuries of red-ink numbers), all the required powers of $\frac{a}{r}$ are formed by the binomial formula:

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$$1 + n\left(\frac{a}{r} - 1\right) + \frac{n(n-1)}{2} \cdot \left(\frac{a}{r} - 1\right)^2 + \text{etc.}$$

When powers of Latitude have been formed by successive multiplication by the series for Latitude, the expression for sine $\overline{\text{Latitude}}|$ and the expressions for the various powers of cosine $|\overline{\text{Latitude}}|$ are formed by means of the appropriate formulæ in powers of Latitude.

6. For the formation of $\frac{dv}{dt}$, it is to be remarked that every argument is formed by the sum of multiples of four simple arguments, namely,

D = Excess of Moon's mean longitude above Sun's mean longitude.

f = Moon's mean argument of latitude.

l = Moon's mean anomaly.

L = Sun's mean anomaly.

The proportion of the daily increase of each of these arguments to the daily increase of the Moon's mean longitude is taken as adopted or found by Delaunay, and the unit of time is supposed to be such that the Moon's mean motion in longitude = 1. Thus, for the term

$$\sin\, \left|\overline{2\,\mathrm{D} + 2f + l - \mathrm{L}}\right|,$$

the sum of the proportions of daily increases of terms under the bracket to daily increase of Moon's mean longitude is found to be 4·7751; therefore

$$\frac{d}{dt}\sin\,\left|\overline{2\,\mathrm{D} + 2f + l - \mathrm{L}}\right|$$

$$= \cos\,\left|\overline{2\,\mathrm{D} + 2f + l - \mathrm{L}}\right| \times 4{\cdot}7751 \times \frac{d}{dt}(\text{daily increase of Moon's mean longitude})$$

$$= 4{\cdot}7751 \times \cos\,\left|\overline{2\,\mathrm{D} + 2f + l - \mathrm{L}}\right|.$$

The corresponding term in $\frac{dv}{dt}$ is found by multiplying this quantity by the coefficient of the corresponding term in v. Succeeding differentiations are effected on the same principle.

7. The methods explained to this point apply to the expansion of functions on the basis of elements assumed by Delaunay, as representing the Moon's motion at a standard modern year (say 1900); without any consideration of the corrections which are due to the coefficients in those expansions, in order to produce a correct theory, on the supposition that the elements of the Earth's orbit round the Sun are stable; and without any consideration of the corrections which are due to the slow changes of the

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elements of the Earth's orbit, which produce secular terms in the Moon's motions. Notice of these corrections will be taken hereafter. At present I may with propriety record the progress made in the limited expansions to which I have first alluded.

8. The following quantities (each expanded into a series of numerous terms with numerical coefficients and periodical trigonometrical functions) are computed, all in duplicate, and some of the earliest in triplicate.

Subject of Expansion.	Red-ink Nos.
Modified equivalent of	
$\frac{a}{r}$ and $\frac{a}{r} - 1$	1— 100
$\left(\frac{a}{r} - 1\right)^2$	201— 348
$\left(\frac{a}{r} - 1\right)^3$	401— 531
$\left(\frac{a}{r} - 1\right)^4$	601— 725
$\left(\frac{a}{r} - 1\right)^5$	801— 826
$\left(\frac{r}{a}\right)^4$	1001—1148
$\left(\frac{r}{a}\right)^3$	1201—1348
$\left(\frac{r}{a}\right)^2$	1401—1548
$\frac{r}{a}$	1601—1748
$\frac{a}{r}$ (above)	1— 100
$\left(\frac{a}{r}\right)^2$	1801—1948
$\left(\frac{a}{r}\right)^3$	2001—2148
$\left(\frac{a}{r}\right)^4$	2201—2348
Equivalent for Latitude	2401—2542
$(\text{Latitude})^2$	2601—2726
$(\text{Latitude})^3$	2801—2959
$(\text{Latitude})^4$	3001—3128
$(\text{Latitude})^5$	3201—3246

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Subject of Expansion.	Red-ink Nos.
$(\text{Latitude})^6$	3401—3434
$\sin \boxed{1}$	3601—3754
$\cos \boxed{1}$	3801—3927
$(\cos \boxed{1})^2$	4001—4127
$(\cos \boxed{1})^3$	4201—4327
$(\cos \boxed{1})^4$	4401—4527
$\cos \boxed{1} \cdot \frac{r}{a}$	4601—4790
$\left(\cos \boxed{1} \cdot \frac{r}{a}\right)^2$	4801—4995
Equivalent for Longitude	5001—5182
$\frac{dv}{dt}$	5201—5382
$\frac{dv}{dt} \cdot \left(\cos \boxed{1} \cdot \frac{r}{a}\right)^2$	5401—5645
$\left(\frac{dv}{dt}\right)^2 \cdot \left(\cos \boxed{1} \cdot \frac{r}{a}\right)^2$	5701—5945
$\frac{d}{dt}\left(\cos \boxed{1} \cdot \frac{r}{a}\right)$	6001—6191
$\left\{\frac{d}{dt}\left(\cos \boxed{1} \cdot \frac{r}{a}\right)\right\}^2$	6201—6450
(Velocity)², or sum of last quantity + last but two	6501—6750
$\frac{d}{dt}(\text{Velocity})^2$	6801—7050

(The velocity here mentioned is the resolved velocity parallel to the plane of the Earth's orbit.)

The duplicate calculation of the quantity last recorded is not yet completed.

The calculation of the following quantities is not yet begun :—

$$\left(\frac{a}{r}\right)^3 \cdot \frac{d}{dt}\left\{\left(\cos \boxed{1} \cdot \frac{r}{a}\right)\right\},$$

$$\frac{r}{a} \cdot \sin \boxed{1},$$

$$\frac{d^2}{dt^2}\left(\frac{r}{a} \cdot \sin \boxed{1}\right),$$

$$\left(\frac{a}{r}\right)^2 \cdot \sin \boxed{1} \cdot$$

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These complete the list of all the quantities not multiplied by the disturbing force which enter into the Equations of the Lunar Theory.

Some of the computed numbers, applying to equations of long period, may require further revision.

9. I now proceed to remark on the state of the "symbolical corrections" mentioned in article (2*) of my former paper, on which the corrections of Delaunay's coefficients, as depending on the elements of the Earth's orbit in a given year (say 1900), are based; and from which the secular equations of the Moon's motion, depending on the slow changes of those elements, are to be deduced. For the present, I limit my details to the symbolical corrections of the coefficients of the periodic terms. The notation used to express these symbolical variations is, the red-ink number defining the coefficient (as above described) surrounded by a ring. This applies to every coefficient in the first instance, and to every subsequent coefficient, except where, by virtue of multiplication by small numbers, it is made too small to possess any value. Thus the series for $\frac{a}{r} - 1$ is expressed by

$$\left(\text{red-ink } 2 + \textcircled{2}\right) . \cos \boxed{l} + \left(\text{red-ink } 3 + \textcircled{3}\right) . \cos \boxed{2 D - l} + \text{etc.};$$

the series for the longitude-terms added to mean longitude is

$$\left(\text{red-ink } 5002 + \textcircled{5002}\right) . \sin \boxed{l} + \left(\text{red-ink } 5003 + \textcircled{5003}\right) . \sin \boxed{2 D - l} + \text{etc.};$$

the series for latitude is

$$\left(\text{red-ink } 2401 + \textcircled{2401}\right) . \sin \boxed{f} + \left(\text{red-ink } 2402 + \textcircled{2402}\right) . \sin \boxed{f + l} + \text{etc.},$$

where "red-ink 2," "red-ink 3," "red-ink 5002," "red-ink 2401," &c., are numerical quantities, adopted or computed as above-mentioned; but

$$\textcircled{2}, \ \textcircled{3}, \ \textcircled{5002}, \ \textcircled{2401}, \ \text{etc.}$$

are still preserved in the symbolic form.

10. As the further treatment of these symbolical terms tends to increase their number and the complexity of their combinations, it is necessary from the beginning to adopt some rule as to the anticipated magnitude of the corrections which Delaunay's coefficients may require. Now I think I may safely assume that no correction greater than 2″·00, or 100 of my notation, can result from the investigation. In this case, it is useless to retain any product by a number smaller than 0·01, or (with allowance for turning of the figure) smaller than 0·005, or smaller than

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50000 of my notation. In the improbable case of an error ultimately showing itself with magnitude greater than 2″·00, the only effect of our limitation would be, that in the equations there is occasionally a loss of some quantity amounting numerically to 1 in the last figure.

11. The application of these considerations will be best understood from an example. The first instance of multiplication is that of the formation of

$$\left(\frac{a}{r}-1\right)^2 \text{ by multiplying } \left(\frac{a}{r}-1\right) \text{ by } \left(\frac{a}{r}-1\right);$$

or

$$\left(r.i.\ 2+\textcircled{2}\right).\cos\boxed{l}+\left(r.i.\ 3+\textcircled{3}\right).\cos\boxed{2\mathrm{D}-l}+\left(r.i.\ 4+\textcircled{4}\right).\cos\boxed{2\mathrm{D}}$$
$$+\left(r.i.\ 5+\textcircled{5}\right).\cos\boxed{2\,l}+\text{etc.};$$

multiplied by

$$\left(r.i.\ 2+\textcircled{2}\right).\cos\boxed{l}+\left(r.i.\ 3+\textcircled{3}\right).\cos\boxed{2\mathrm{D}-l}+\left(r.i.\ 4+\textcircled{4}\right).\cos\boxed{2\mathrm{D}}$$
$$+\left(r.i.\ 5+\textcircled{5}\right).\cos\boxed{2\,l}+\text{etc.},$$

which will produce

$$\left(r.i.\ 201+(\overline{\underline{201}})\right).\cos\boxed{0}+\left(r.i.\ 202+(\overline{\underline{202}})\right).\cos\boxed{l}+\left(r.i.\ 203+(\overline{\underline{203}})\right).\cos\boxed{2\mathrm{D}-l}$$
$$+\left(r.i.\ 204+(\overline{\underline{204}})\right).\cos\boxed{2\mathrm{D}}+\left(r.i.\ 205+(\overline{\underline{205}})\right).\cos\boxed{2\,l}+\text{etc.}$$

Now, cos $\boxed{0}$ is produced by the combination of cos $\boxed{l}$ with cos $\boxed{l}$, of cos $\boxed{2\mathrm{D}-l}$ with cos $\boxed{2\mathrm{D}-l}$, of cos $\boxed{2\mathrm{D}}$ with cos $\boxed{2\mathrm{D}}$, of cos $\boxed{2\,l}$ with cos $\boxed{2\,l}$, etc., and its coefficient, or $r.i.\ 201+(\overline{\underline{201}})$, will therefore be

$$\frac{1}{2}(r.i.\ 2)^2+r.i.\ 2\times\textcircled{2},$$
$$+\frac{1}{2}(r.i.\ 3)^2+r.i.\ 3\times\textcircled{3},$$
$$+\frac{1}{2}(r.i.\ 4)^2+r.i.\ 4\times\textcircled{4},$$
$$+\frac{1}{2}(r.i.\ 5)^2+r.i.\ 5\times\textcircled{5},$$
$$+\text{etc.}$$

Now $r.i.\ 2=+545094$; $r.i.\ 3=+99812$; $r.i.4=-82321$;

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r.i. 5 = +29794. Here then it appears that *r.i.* 2, *r.i.* 3, *r.i.* 4, are above our limit; but *r.i.* 5 and all succeeding numbers (which are still smaller) are below it. The first three, then, of the products of symbolical variations just given, are the only ones to be retained for $(\overline{201})$. Striking off the last five figures we have

$$(\overline{201}) = +5 \times \textcircled{2} + 1 \times \textcircled{3} + 1 \times \textcircled{4},$$

where the unit of the factors is $\frac{\text{unity}}{100}$, or 10^{-2}.

Now consider the factor of $\cos \lceil l \rceil$ in the term

$$\left(r.i.\ 202 + (\overline{202})\right) . \cos \lceil l \rceil .$$

Here, $\cos \lceil l \rceil$ may be produced by the combination of $\cos \lceil l \rceil$ with $\cos \lceil 2l \rceil$, of $\cos \lceil 2\,\mathrm{D} - l \rceil$ with $\cos \lceil 2\,\mathrm{D} \rceil$, of $\cos \lceil 2\,\mathrm{D} \rceil$ with $\cos \lceil 2\,\mathrm{D} - l \rceil$, of $\cos \lceil 2\,\mathrm{D} - l \rceil$ with $\cos \lceil 2\,\mathrm{D} - 2l \rceil$, and of others which are at once seen to be insensible. Hence, the value of $(r.i.\ 202 + (\overline{202}))$ is

$$+ \left(r.i.\ 2 + \textcircled{2}\right) \times \left(r.i.\ 5 + \textcircled{5}\right),$$

$$+ \left(r.i.\ 3 + \textcircled{3}\right) \times \left(r.i.\ 4 + \textcircled{4}\right),$$

$$+ \left(r.i.\ 4 + \textcircled{4}\right) \times \left(r.i.\ 6 + \textcircled{6}\right),$$

$$+ \left(r.i.\ 3 + \textcircled{3}\right) \times \left(r.i.\ 19 + \textcircled{19}\right);$$

and the value of $(\overline{202})$ is

$$+ r.i.\ 2 \times \textcircled{5} \qquad + r.i.\ 5 \times \textcircled{2},$$

$$+ r.i.\ 3 \times \textcircled{4} \qquad + r.i.\ 4 \times \textcircled{3},$$

$$+ r.i.\ 4 \times \textcircled{6} \qquad + r.i.\ 6 \times \textcircled{4},$$

$$+ r.i.\ 3 \times \textcircled{19} \qquad + r.i.\ 19 \times \textcircled{3}.$$

The only factors which are sensible are *r. i.* 2, *r. i.* 3, *r. i.* 4; and therefore

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$$\left(\overline{\underline{202}}\right) = +5 \times \textcircled{5} + 1 \times \textcircled{4} + 1 \times \textcircled{3} + 1 \times \textcircled{6} + 1 \times \textcircled{19},$$

where (as before) the unit of the factors is $\frac{\text{unity}}{100}$, or 10^{-2}.

When the series for $\left(\frac{a}{r} - 1\right)^2$ is multiplied by another series, a similar process must be used for expressing the symbolical variations of the coefficients then newly formed, in terms of $(\overline{\underline{201}})$, $(\overline{\underline{202}})$, etc.; and of the variations of terms of the other series; then the values found above are to be substituted for $(\overline{\underline{201}})$, $(\overline{\underline{202}})$, etc., and finally the symbolical variation of every coefficient, without exception, will be found in terms of

$$\textcircled{2},\ \textcircled{3},\ \textcircled{4},\ \text{etc.},\ (\overline{\underline{2401}}),\ (\overline{\underline{2402}}),\ \text{etc., and}\ (\overline{\underline{5001}}),\ (\overline{\underline{5002}}),\ \text{etc.}$$

12. The symbolical variations are formed as far as

$$\left(\cos\,|\overline{l}| \times \frac{r}{a}\right)^2$$

in the table of article 8, but may require further revision.

13. To complete the exhibition of symbolical variations, it is necessary to prepare the variations produced by the symbolical variations of the arguments. Thus, for the term

$$r.i.\ 8 \times \cos\,|\overline{2\,\mathrm{D}-l-\mathrm{L}}|,$$

the complete variation is

$$\textcircled{8}\,.\cos\,|\overline{2\,\mathrm{D}-l-\mathrm{L}}| - r.i.\ 8\,.\sin\,|\overline{2\,\mathrm{D}-l-\mathrm{L}}| \times \left(\overline{\underline{2\,\mathrm{D}-l-\mathrm{L}}}\right).$$

In the ordinary process of multiplying series by series, this operation is remarkably simple; it is not necessary to go step by step, as in the preparation of the variations of the coefficients in article 11; but the variation depending on the argument will be found correctly from the argument standing at the end of the operations (as will easily be seen by trying a step-by-step operation). A trifling complexity is introduced in these terms which bear the sign of differentiation, because the symbolical variations themselves are subject to differentiation.

14. As the variation of arguments produces its effect in the grand final equation only where it is multiplied by a coefficient which is usually small, and as it is particularly in these variations that we must look for the terms which rise to importance in the shape of secular terms, we must be prepared to accept much larger values for the magnitude of the terms to be produced by these symbolical variations of arguments. I have not yet decided on the limit to be adopted.

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15. No step of calculation has yet been made in reference to the symbolical variation of arguments.

16. No calculation is yet made for that side of the final equation which contains the disturbing force as a factor.

In any future Report which it may appear desirable to place before the Society, there will be no occasion to repeat any part of the explanation of methods given here, and it may be anticipated that the statement of the step which the operations may have reached will be very brief.

Royal Observatory, Greenwich,
1875, *February* 6.

Observations of the Satellites of the Planet Uranus, made at Birr Castle, during the years 1872, 1873, *and* 1874.

By the Earl of Rosse.

The following observations of the Satellites of the planet *Uranus*, made with the reflector of six-feet aperture at Birr Castle, though for several reasons not as satisfactory as might be desired and might be obtained with that instrument, may perhaps be of some value as a contribution to knowledge in a department of astronomy in which so few have as yet worked.

The shortness of the time (40 minutes) available each night for the observation of the planet with the 6-foot instrument, the atmospheric disturbance so often a source of annoyance in using so large an aperture, the unsuitability of the micrometer for the work, and to a certain extent also the almost unavoidable deterioration of the polish of the mirrors in the course of working, which probably increased the diffused light round the planet, combined to detract from the accuracy of the observations, and to render the detection of the inner satellites difficult.

My late assistant, Dr. Ralph Copeland, has compared the observed places of *Oberon* and *Titania* with those which he has calculated from Dr. von Asten's definite circular Elements given in Vol. xviii., No. 5, page 26, of the *Memoirs* of the Imperial Academy of Sciences of St. Petersburg. The differences between calculation and observation are given in the column headed (C. − O.). The observations made by Dr. Copeland and myself are distinguished respectively by the letter C. or R.